

# Lower Thames Crossing

## 6.3. Environmental Statement Appendices

### Appendix 9.1 Assessment of ground-borne noise and vibration, and underwater noise from the tunnel boring machine at marine receptors

APFP Regulation 5(2)(a)

Infrastructure Planning (Applications:  
Prescribed Forms and Procedure)  
Regulations 2009

Volume 6

**DATE:** October 2022

Planning Inspectorate Scheme Ref: TR010032  
Application Document Ref: TR010032/APP/6.1

**VERSION:** 1.0

## Lower Thames Crossing

### 6.3. Environmental Statement Appendices Appendix 9.1 TBM NW Assessment

#### List of contents

	Page number
<b>1 Introduction</b> .....	<b>1</b>
<b>2 Background</b> .....	<b>2</b>
<b>3 Prediction of vibration and noise from tunnel boring machines</b> .....	<b>3</b>
<b>4 Numerical modelling</b> .....	<b>4</b>
<b>5 Modelling assumptions used</b> .....	<b>5</b>
<b>6 Significance criteria</b> .....	<b>7</b>
6.1 Significance criteria – fish .....	7
6.2 Marine mammals .....	10
<b>7 Results</b> .....	<b>12</b>
7.1 Underwater sound .....	12
<b>8 Conclusions</b> .....	<b>13</b>
<b>References</b> .....	<b>14</b>
<b>Annex A The FINDWAVE® model</b> .....	<b>15</b>

## List of plates

	<b>Page number</b>
Plate 3.1 Cutaway view of a large diameter TBM (Herrenknecht) .....	3
Plate 5.1 Generic long section through a typical TBM model.....	5
Plate 5.2 Generic cross-section through a typical TBM model.....	6
Plate 6.1 Examples of fish hearing thresholds .....	9
Plate 6.2 Auditory weighting functions for low-frequency (LF) cetaceans, mid-frequency (MF) cetaceans, high-frequency (HF) cetaceans, and phocid pinnipeds (PW) (underwater) (from NMFS, 2016) .....	10
Plate 7.1 Worst-case spectrum of underwater sound level due to the TBM.....	12

## List of tables

	<b>Page number</b>
Table 6.1 Criteria for assessing mortality and potentially mortal injury, recoverable injury and temporary threshold shift in species of fish (Popper <i>et al.</i> , 2014) .....	8
Table 6.2 Marine mammal hearing groups (from NMFS, 2016).....	10
Table 6.3 Criteria for assessing auditory injury and TTS in marine mammals (NMFS, 2016) .....	11
Table 6.4 Criteria for assessing disturbance/behavioural reaction in marine mammals ....	11

# 1 Introduction

- 1.1.1 Rupert Taylor Ltd was instructed by the Project to carry out a study of the likely level of ground-borne noise and vibration from the excavation and construction of the proposed Lower Thames Crossing tunnels. The principal activities involved are the tunnel drive using a tunnel boring machine (TBM), and the construction of cross-passages between the two tunnels. Between the tunnel portals the potential sensitive receptors are sub-aquatic species in the River Thames. This report gives the results of a study carried out to predict the likely levels of underwater noise above the alignment. Of the two principal activities the TBM produces an effect greater than the works involved in the construction of cross passages. Consequently, it is only necessary to consider effects during the passage of the TBM. These are assessed against relevant criteria.
- 1.1.2 Models were created to study the propagation of vibration from the tunnel face, with the TBM operating in the soils which are likely to occur along the alignment. The output of the modelling is an indication of likely ground-borne vibration and associated underwater noise.

## 2 Background

- 2.1.1 The factors which influence the generation and propagation of vibration and ground-borne noise from TBMs are primarily the amount of energy required to cut the soil or rock and the propagation characteristics of the soil. Rotational speed, cutter head type and face pressure have a much smaller effect. The energy requirement is a function of the tunnel diameter and the operating characteristics of the machine. The proposed Lower Thames Crossing tunnels are likely to be approximately 16.5m diameter. The diameter of the Jubilee Line Extension TBM face was 4.9m, Dublin Port Tunnel was 11.8m, Crossrail was 7.1m, High Speed 1 London tunnels were 8.11m diameter and the Silvertown Tunnel design is designed for 12.5m diameter.
- 2.1.2 Soil type is a major influence, with London Clay being soft enough for the main noise from the TBM to be machinery on the TBM. At the other extreme, excavating through rock generates a large amount of noise and vibration due to the cutting effect itself. Previous tunnels in the UK and Ireland were in a variety of lithologies. Dublin is Carboniferous limestone below glacial till. The tunnels in London are in London Clay Formation, gravel, Lambeth Group, chalk and Thanet Formation sands. The Silvertown tunnel will be bored through clay, gravel and sand. The Project tunnels will be driven through weak chalk under the River Thames, below layers of river terrace deposits and alluvium, including some clay and gravel.
- 2.1.3 The Lower Thames Crossing TBMs are likely to be slurry machines, although from a vibration point of view there is little difference between slurry and earth pressure-balance machines for the same soil conditions.

## 3 Prediction of vibration and noise from tunnel boring machines

- 3.1.1 The prediction of vibration and ground-borne noise from tunnel boring machines has to begin with measured field data which are used to calibrate the output of a model for predicting the spatial spread of the vibration (which in turn may also cause ground-borne noise). In stiff or hard soils the source is concentrated at the cutter face. In soft soil, ground-borne noise may be radiated from the entire length of the TBM, which can reach lengths of 100m or so (see Plate 3.1).

**Plate 3.1 Cutaway view of a large diameter TBM (Herrenknecht)**



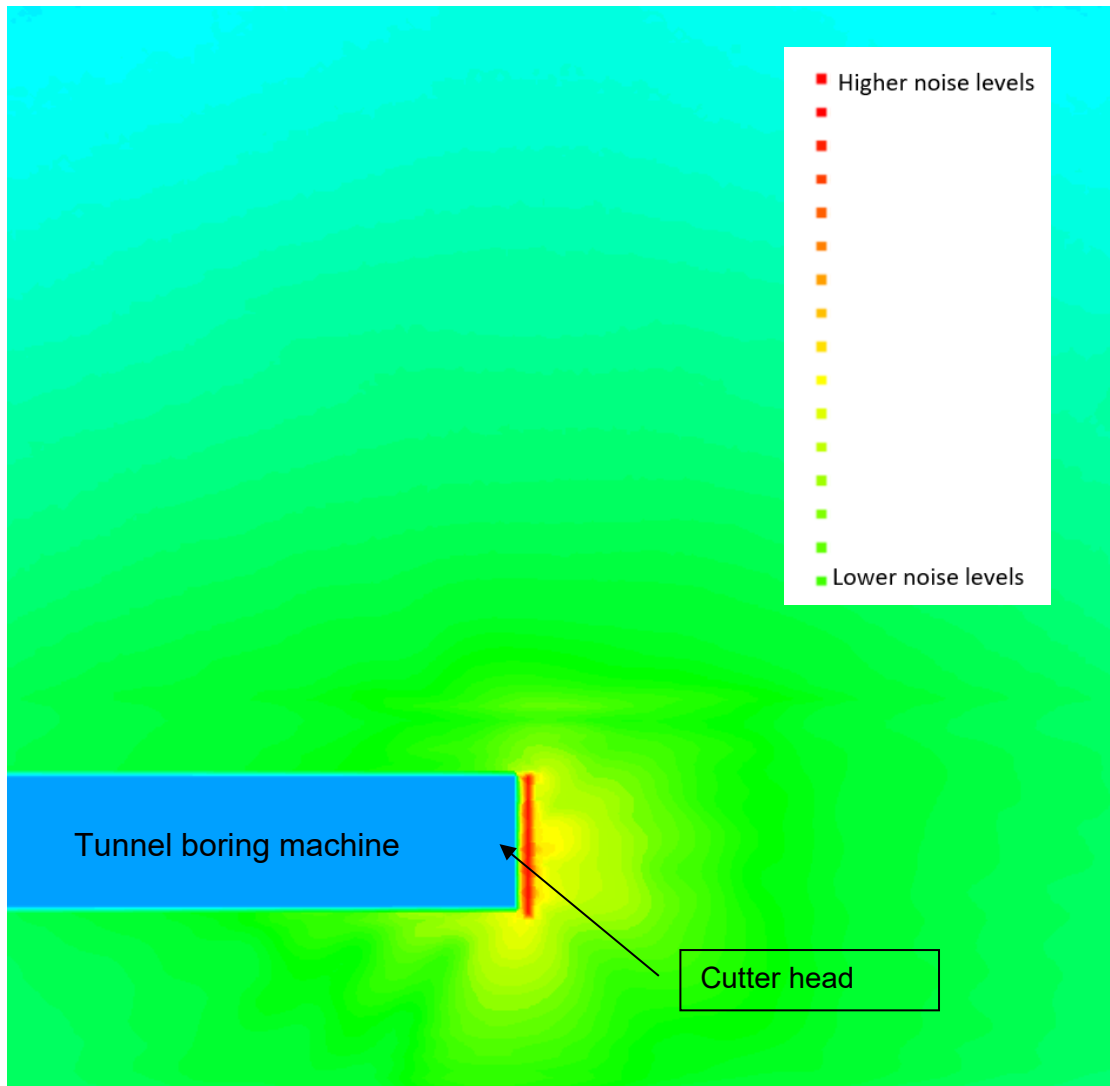
## 4 Numerical modelling

- 4.1.1 The predictions of vibration and ground-borne noise were carried out using the Rupert Taylor Finite Difference Time Domain model, FINDWAVE.
- 4.1.2 The model used for this study predicts, in the time domain, the three-dimensional vibration velocity of the tunnel face and surrounding lithology. The time-domain results are transformed into the frequency domain to give 1/3 octave frequency spectra, and overall sound levels in dB(A) and vibration units.
- 4.1.3 The model has been calibrated by using the model to predict vibration from the Crossrail TBM, and back-fitting the results from field measurements obtained during the tunnel drive. This approach was previously adopted in the Environmental Assessment of the Silvertown Tunnel project leading to the grant of its Development Consent Order.
- 4.1.4 The approach has been to set up a group of generic models in a selection of soil conditions and produce cross-sectional plots of vectored soil velocity from which, subject to the application of transfer functions to buildings, ground surface predictions can be made.
- 4.1.5 FINDWAVE is a finite-difference time-domain numerical model for computing the propagation of waves in elastic media. Full details of the model are given in Annex A. The excitation is provided from a random array of impulses applied to the tunnel face. The model predicts, in the time domain, the dynamic behaviour the medium surrounding the tunnel face.
- 4.1.6 The model has a cell size of 200mm in the lateral and vertical directions, and 200mm in the longitudinal direction (along the tunnel). A time step of 1/131072 seconds was used. The model was run for a time period of 1 second. Output from the model consists of time series of the velocity of transverse and longitudinal sections through the model, which are subjected to frequency transformation and expressed as 1/3 octave band spectra.

## 5 Modelling assumptions used

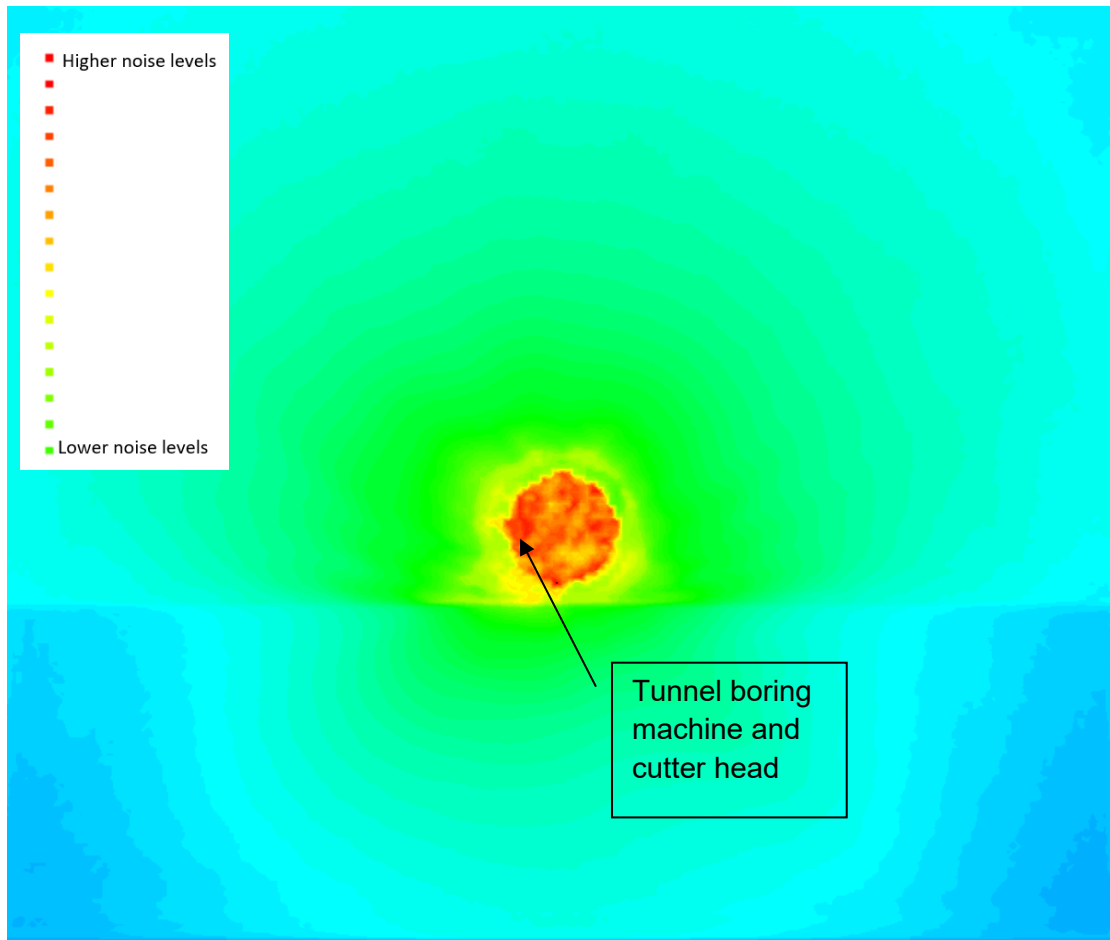
- 5.1.1 The geotechnical data assumed in the modelling was taken from various factual and interpretative sources of information available at the time.
- 5.1.2 The tunnel behind the TBM was assumed to be lined with concrete.
- 5.1.3 Examples of generic outputs from a typical TBM model are shown in Plate 5.1 and Plate 5.2.

**Plate 5.1 Generic long section through a typical TBM model**





**Plate 5.2 Generic cross-section through a typical TBM model**



## 6 Significance criteria

### 6.1 Significance criteria – fish

- 6.1.1 Underwater sound is expressed using a different decibel scale to that used for airborne sound, with a reference level of  $1\mu\text{Pa}$  rather than the  $20\mu\text{Pa}$  in the case of airborne sound. Furthermore, the characteristic impedance of water is much greater than that of air, resulting in underwater sound levels being numerically much higher than would be the case if the water were replaced by air.
- 6.1.2 Fish can sense waterborne noise through both auditory and mechanosensory systems (Clark *et al.*, 1996). Hearing threshold curves for various species of fish have been obtained and over the mid-frequency range are typically 75dB re  $1\mu\text{Pa}$  or higher (underwater dB are usually referenced to a pressure of 1 microPascal).
- 6.1.3 Most research has centred on the relationship between sound pressure levels and hearing damage in fish. Sounds that were lower than 180dB (re  $1\mu\text{Pa}$ ) and sounds that were not continuous had no apparent impact on the sensory cells of the ear (Hastings *et al.*, 1996).
- 6.1.4 It is not known what effect noise has in the range between 75 and 180dB, on such matters as the behaviour of fish in, for example, swimming to their spawning grounds. Rivers and oceans are, however, naturally noisy, with heavy rain producing sound pressure levels up to 110dB. Shipping can produce noise levels over 105dB.
- 6.1.5 Fish species also have pressure-sensitive cells that are sensitive to low frequencies (typically 10Hz to 30Hz) and near-field pressure changes, limited to an area immediately surrounding the fish. This allows the fish to detect the presence of other fish in close proximity (such as in schooling behaviour) or assists in predatory avoidance. The sensitivity of the pressure-sensitive cells, however, is at an insufficient level to detect the TBM operation.
- 6.1.6 Fish are known to have complex and diverse inner ear structures with a broad range of hearing sensitivities, generally weighted towards lower frequencies. Some species, such as some members of the herring family, can hear across a wider range of frequencies and quieter sounds – these species are sometimes referred to as ‘hearing specialists’. Others, particularly flatfish, e.g. dab and plaice, hear over a narrower range of frequencies and have much lower sensitivities to sound (they have no swim bladders); salmon and trout also fall into this latter group, which are sometimes referred to as ‘hearing generalists’. Some examples of fish hearing thresholds (Nedwell *et al.*, 2004) are given in Plate 6.1.
- 6.1.7 Popper *et al.* (2014) groups species of fish according to whether they possess a swim bladder, and whether it is involved in its hearing. This guidance gives specific criteria, as both  $\text{SPL}_{\text{peak}}$  and  $\text{SEL}_{\text{cum}}$  values, for a variety of noise sources. The modelled criteria are summarised in Table 6.1.  $\text{SEL}_{\text{cum}}$  is defined as the sound exposure level over a number of individual impulsive sound exposures and is calculated as the log sum of the squared sound pressure of

the individual events. It is primarily applicable to impulsive sound from, for example, percussive piling, which is outside the scope of this report. Underwater noise from the TBM will not be impulsive. TBM noise is broadly continuous during the time that the cutter face is being thrust forward, and therefore  $SEL_{cum}$  levels can be obtained from  $SPL_{peak}$  levels by adding  $10 \log_{10} T$  where  $T$  is the number of seconds for which the sound is heard.

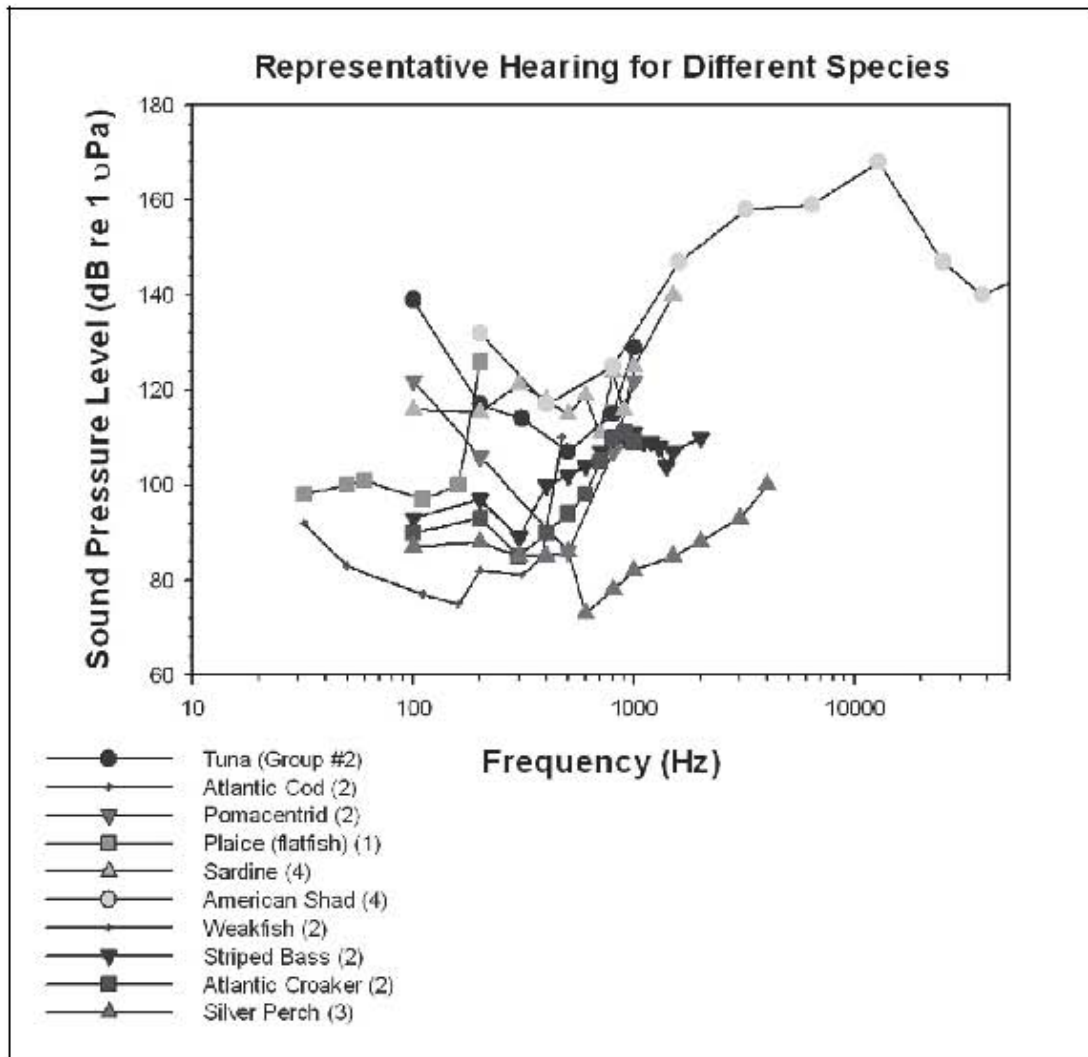
**Table 6.1 Criteria for assessing mortality and potentially mortal injury, recoverable injury and temporary threshold shift in species of fish (Popper *et al.*, 2014)**

Type of animal	Mortality and potential mortal injury	Impairment	
		Recoverable injury	Temporary threshold shift
Fish: no swim bladder	>219dB $SEL_{cum}$ or >213dB $SPL_{peak}$	>216dB $SEL_{cum}$ or >213dB $SPL_{peak}$	>186dB $SEL_{cum}$
Fish: swim bladder is not involved in hearing	210dB $SEL_{cum}$ or >207dB $SPL_{peak}$	203dB $SEL_{cum}$ or >207dB $SPL_{peak}$	>186dB $SEL_{cum}$
Fish: swim bladder involved in hearing	207dB $SEL_{cum}$ or >207dB $SPL_{peak}$	203dB $SEL_{cum}$ or >207dB $SPL_{peak}$	186dB $SEL_{cum}$

\* $SPL_{peak}$ - single, unweighted peak criteria /  $SEL_{cum}$ - cumulative (i.e. more than a single sound impulse), weighted criteria

- 6.1.8 Popper *et al.* also consider behavioural effects in fish, which are defined as ‘substantial change in behaviour for the animals exposed to a sound. This may include long-term changes in behaviour and distribution, such as moving from preferred sites for feeding and reproduction, or alteration of migration patterns.’
- 6.1.9 The Popper *et al.* (2014) guidelines conclude that there is insufficient data available to apply quantitative thresholds for behavioural effects on fish.

**Plate 6.1 Examples of fish hearing thresholds**



6.1.10 The Thames estuary is considered an important habitat for a variety of fish species. There is a mixed community of fish within the estuary driven by the seasonal movement of the various species, ranging from fully marine to freshwater species with no estuarine requirements.

6.1.11 The community includes:

- a. Freshwater species able to tolerate saline water such as perch, bream and dace;
- b. Species of conservation importance that either migrate through the estuary such as salmon, sea trout, shad, lamprey, smelt and eel;
- c. Common estuarine species such as gobies, sprat, bass, flounder and sole;
- d. Species utilising the estuary for nursery areas such as sole, bass and flounder; and
- e. Commercially important species such as herring, mullet, dab, cod and whiting.

6.1.12 The results reported show that underwater sound levels caused by the TBM are likely to be well above the threshold of hearing of fish, and also likely to be well above ambient sound levels, although significantly below hearing damage thresholds. The effect that this can be assumed to have on the behaviour of fish will be that they tend to swim away from the source of the sound.

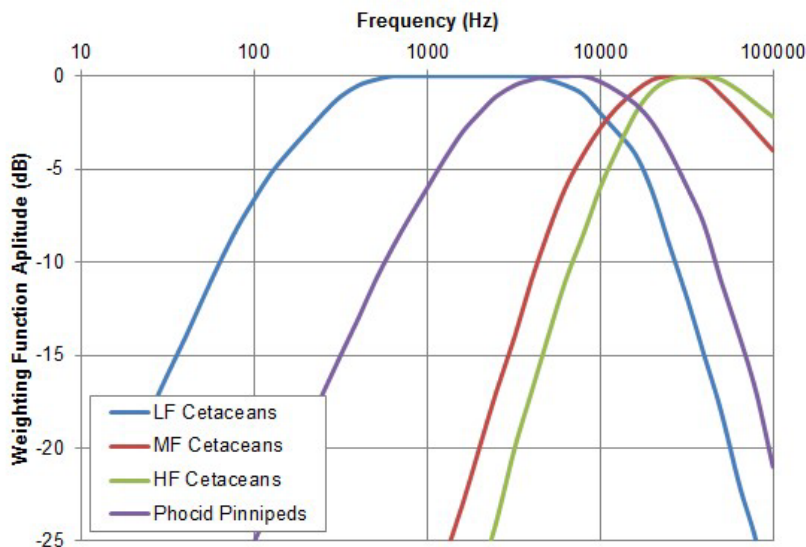
## 6.2 Marine mammals

6.2.1 The National Marine Fisheries Service (NMFS) (2016) guidance for marine mammals covers underwater noise and its effects. The guidance groups marine mammals into functional hearing groups and applies filters to the unweighted noise to approximate the hearing sensitivity of the receptor. The hearing groups given in the NMFS (2016) guidance are summarised in Table 6.2 and Plate 6.2. A further group for otariid pinnipeds is also given in the guidance for sea lions and fur seals but has not been used in this study as those species are not found in this region.

**Table 6.2 Marine mammal hearing groups (from NMFS, 2016)**

Hearing group	Example species	Generalised hearing range
Low-frequency (LF) cetaceans	Baleen whales	7Hz to 35kHz
Mid-frequency (MF) cetaceans	Dolphins, toothed whales, beaked whales, bottlenose whales (including bottlenose dolphin)	150Hz to 160kHz
High-frequency (HF) cetaceans	True porpoises (including harbour porpoise)	275Hz to 160kHz
Phocid pinnipeds (PW) (underwater)	True seals (including harbour seal)	50Hz to 86kHz

**Plate 6.2 Auditory weighting functions for low-frequency (LF) cetaceans, mid-frequency (MF) cetaceans, high-frequency (HF) cetaceans, and phocid pinnipeds (PW) (underwater) (from NMFS, 2016)**



6.2.2 Table 6.3 presents single strike, unweighted peak criteria ( $SPL_{peak}$ ) and cumulative (i.e. more than a single sound impulse), weighted sound exposure criteria ( $SEL_{cum}$ ) for both permanent threshold shift (PTS), where unrecoverable hearing damage may occur, and temporary threshold shift (TTS), where a temporary reduction in hearing sensitivity may occur in individual receptors.

**Table 6.3 Criteria for assessing auditory injury and TTS in marine mammals (NMFS, 2016)**

NMFS (2016)	Unweighted $SPL_{peak}$ (dB re 1 $\mu$ Pa)	Weighted $SEL_{cum}$ (dB re 1 $\mu$ Pa <sup>2</sup> s)	
	Auditory injury (PTS)	Auditory injury (PTS)	Temporary threshold shift)
Low-frequency (LF) cetaceans	219	183	168
Mid-frequency (MF) cetaceans	230	185	170
High-frequency (HF) cetaceans	202	155	140
Phocid pinnipeds (PW) (underwater)	218	185	170

6.2.3 Where  $SEL_{cum}$  are required, a fleeing animal model has been used. This assumes that the animal exposed to high noise levels will swim away from the noise source. For this, a constant fleeing speed of 1.5ms<sup>-1</sup> has been assumed, which is a cruising speed for a harbour porpoise (Otani *et al.*, 2000). These are considered ‘worst-case’ as marine mammals are expected to be able to swim much faster under stress conditions. The model assumes that a fleeing receptor stops if it reaches the coast before the noise exposure ends. The PTS and TTS criteria and results for low-frequency cetaceans have been included for completeness, although it is understood that species in this functional group are not considered a concern for the Project.

6.2.4 Criteria for disturbance or behavioural reaction effects in marine mammals are in development by NMFS (2016). For this assessment, thresholds as single strike SEL have been derived from data presented in Southall *et al.* (2007) for mid frequency and Lucke *et al.* (2009) for high frequency cetaceans, as presented in Table 6.4. The disturbance threshold for seals is as per TTS. Criteria have not been presented for low frequency cetaceans, as these species are not generally present in the area.

**Table 6.4 Criteria for assessing disturbance/behavioural reaction in marine mammals**

Hearing group	Behavioural reaction SEL re 1 $\mu$ Pa <sup>2</sup> s
Mid-frequency (MF) cetaceans	160dB
High-frequency (HF) cetaceans	145dB

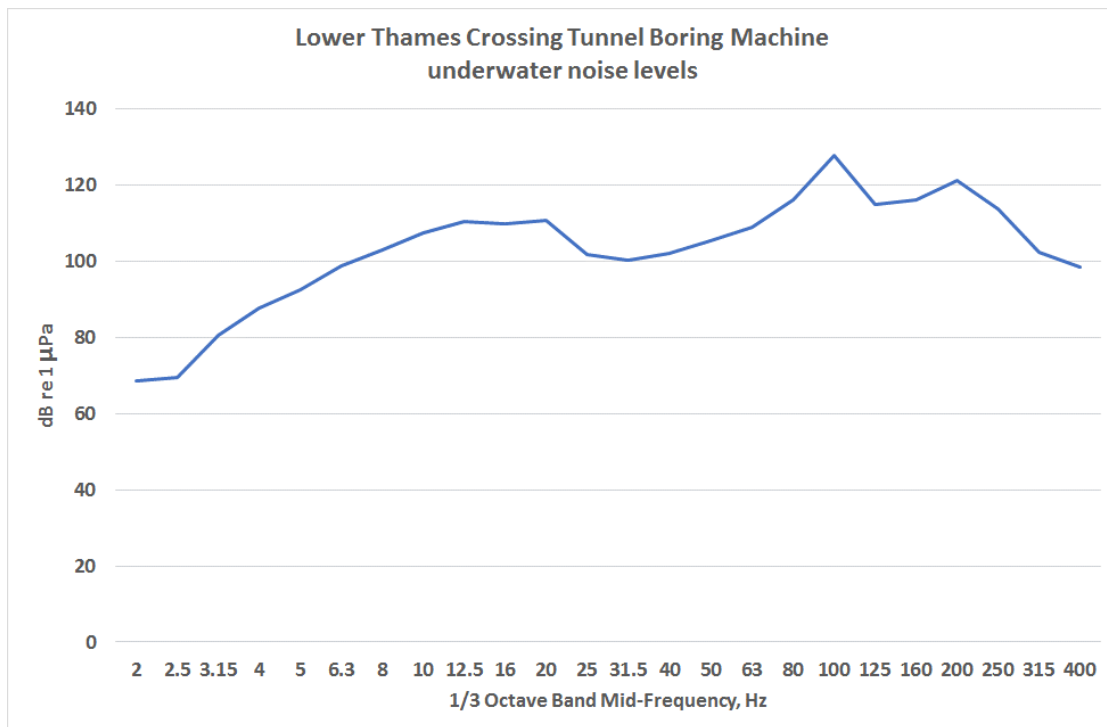
## 7 Results

### 7.1 Underwater sound

7.1.1 The full results of the underwater noise prediction are shown in contour maps for the length of tunnel under the river.

7.1.2 In terms of spectral content, the model shown in Plate 7.1 is for the worst-case location near the TBM, close to the position of the face.

**Plate 7.1 Worst-case spectrum of underwater sound level due to the TBM**



## 8 Conclusions

- 8.1.1 In summary the results were as follows, together with the relevant significance criteria.
- 8.1.2 Underwater sound: 130 dB re 1  $\mu$ Pa, would be well above fish hearing thresholds but well below damage risk threshold of 180 dB.
- 8.1.3 The predicted levels are well above the hearing thresholds of at least some fish species but are well below hearing damage thresholds. The effect of the noise is likely to be temporary and to cause fish to move away from the vicinity of the TBM.
- 8.1.4 With regard to marine mammals, the worst case noise level is below the threshold for temporary threshold shift in the most sensitive group, high-frequency cetaceans. The behavioural reaction threshold would be reached after being present in the worst-case location for approximately half a minute.



## References

- Clark, J.A., Young, J.A., Bart, A. and Zohar, Y. (1996). Physiological effects of infrasonic noise on captive fish. *Journal of the Acoustical Society of America*, 100(4): pp. 2709.
- Hastings, M.C., Popper, A.N., Finneran, J.J. and Lanford, P.J. (1996). Effects of low-frequency underwater sound on hair cells of the inner ear and lateral line of the teleost fish *Astronotus ocellatus*. *Journal of the Acoustical Society of America*, 99(3): pp. 1759–1766.
- Lucke, K., Siebert, U. and Lepper, P.A. (2009). Temporary shift in masked hearing thresholds in a harbor porpoise (*Phocoena phocoena*) after exposure to seismic airgun stimuli. *The Journal of the Acoustical Society of America* 125(6): pp. 4060-70.
- National Marine Fisheries Service (NMFS) (2016). Draft Guidance for Assessing the Effects of Anthropogenic Sound on Marine Mammals: Acoustic Threshold Levels for Onset of Permanent and Temporary Threshold Shifts.
- Nedwell, J.R., Edwards, B., Turnpenny, A.W.H. and Gordon, J. (2004). Fish and Marine Mammal Audiograms: a summary of available information. Prepared by Subacoustech Ltd., Hampshire, UK. Report ref: 534R0214.
- Otani, S., Naito, Y., Kato, A. and Kawamura, A. (2000). Diving behavior and swimming speed of a free-ranging harbor porpoise, *Phocoena phocoena*. *Marine Mammal Science* 16(4): pp. 811 – 814.
- Popper, A.N., Hawkins, A.D., Fay, R.R., Mann, D.A., Bartol, S., Carlson, T.J., Coombs, S., Ellison, W.T., Gentry, R.L., Halvorsen, M.B., Løkkeborg, S., Rogers, P.H., Southall, B.L., Zeddes, D.G. and Tavolga, W.N. (2014). ASA S3/SC1.4 TR-2014 Sound Exposure Guidelines for Fishes and Sea Turtles: a technical report prepared by ANSI-Accredited Standards Committee S3/SC1 and registered with ANSI. SpringerBriefs in Oceanography.
- Southall, B. L., Bowles, A., Ellison, W. and Finneran, J. (2007). Marine mammal noise exposure criteria. *Aquatic Mammals*, 33: pp. 411-521.

## Annex A The FINDWAVE® model

### A.1 Introduction

A.1.1 The wave equation in differential form is as follows

$$(1) \quad \mu \left( \frac{\partial^2 \xi}{\partial x^2} + \frac{\partial^2 \xi}{\partial y^2} + \frac{\partial^2 \xi}{\partial z^2} \right) + (\lambda + \mu) \left( \frac{\partial^2 \xi}{\partial x^2} + \frac{\partial^2 \eta}{\partial x \partial y} + \frac{\partial^2 \zeta}{\partial x \partial z} \right) = \rho \frac{\partial^2 \xi}{\partial t^2}$$

for the x axis, with corresponding equations for the y and z axes, where x, y, z and  $\xi$ ,  $\eta$ ,  $\zeta$  are displacements in three orthogonal axes;  $\lambda$  and  $\mu$  are Lamé constants and  $\rho$  is the density. The Lamé constant  $\mu$  is also known as the shear modulus,  $G$ . The Lamé constant  $\lambda$  is also known as the coefficient of dilatation and is given by

$$\lambda = \frac{2\sigma G}{(1-2\sigma)}$$

where  $\sigma$  is Poisson's ratio.

A.1.2 Equation (1) can be stated in finite difference form by replacing the differential operator with the approximation

$$\frac{\partial \xi}{\partial x} \approx (x[i][j][k] - x[i-1][j][k]) / \Delta x \quad (2)$$

For  $\Delta x \rightarrow 0$ , these two forms are identical.

A.1.3 For a homogeneous, isotropic medium with a finite value for  $\Delta x$ ,  $\Delta y$  and  $\Delta z$ , elastic wave propagation can be computed using the finite difference substitution of equation (2).

A.1.4 Effectively, the process is as follows, for each axis, i, j and k. The example given is for axis i. Each point p(i,j,k) lies at the corner of a rectangular cell and is assigned a mass equal to one-eighth of the sum of the eight contiguous cells as well as a displacement and velocity. The displacement and velocity is interpolated for each intermediate "virtual" point p(i+d,i+d,k+d) where d=0 or 0.5.

- a. Compute pressure gradient.
- b. Compute shear force gradient.
- c. Accelerate p(i,j,k) by  $\Delta v = F / \rho \Delta t$  where  $F$  is the sum of the force 1 & 2 and  $\rho$  is the density assigned to the point and  $v$  is the point velocity.
- d. Displace p(i,j,k) by  $\Delta x = \Delta v * \Delta t$  where  $x$  is the point displacement and  $t$  is one time step.
- e. Repeat from step 1.

- A.1.5 The geometric part of wave propagation is completely represented by this process. Further terms are required to represent damping. Of several possible terms, the inclusion of a coefficient by which the velocity is multiplied produces a loss factor which decreases within increasing frequency (and gives rise to an excess attenuation per unit distance which is independent of frequency). A viscous damping term can be used, by including a force proportional to acceleration multiplied by a coefficient. However, many materials exhibit hysteretic damping, or damping with other types of frequency dependence. To model these effects, it is necessary to include an algorithm which implements Boltzmann's strain history method where

$$s(t) = D_1 \varepsilon(t) - \int_0^{\infty} \varepsilon(t - \Delta t) \varphi(\Delta t) d(\Delta t)$$

where  $\varphi(\Delta t) = \frac{D_2}{\tau} e^{-\Delta t / \tau}$  is an after-effect function,  $D_2$  is a constant and  $\tau$  is a relaxation time.  $D_1$  is a modulus,  $s(t)$  is stress and  $\varepsilon(t)$  is strain. By combining several after-effect functions with different values of  $D_2$  and  $\tau$ , any relationship between loss factor and frequency may be represented. Note that in the frequency domain the integral has a real and imaginary part, with the result that the value of the modulus is reduced by the inclusion of the relaxation terms. Depending on the choice of the constants and relaxation times, the stiffness of a resilient element will be frequency-dependent, and the value of  $D_1$  must be adjusted at the same time that  $D_2$  and  $\tau$  are selected to give the required dynamic stiffness. This method has been implemented in the version of FINDWAVE® used for this study.

## A.2 Boundaries

- A.2.1 For modelling finite objects fully surrounded by space, the boundaries can be represented by assigning zero-valued elastic moduli to the space, provided that the acoustic load of the air in an airspace can be neglected. If radiation into air is to be modelled, or if an infinite or semi-infinite medium such as the ground is required, it is necessary to minimise the effect of reflections from the boundaries. For a train tunnel, where distances to be modelled are small compared with the length of the train, the z-axis boundaries are dealt with by creating a model exactly one rail vehicle (or unit of several coupled rail vehicles) in length, and then connecting the ends of the model together to create an infinitely long train. This is done by copying the cell displacements and velocities from one end of the model to the other end at the end of each time-step.
- A.2.2 For the other boundaries in the x- and y-axes, the potential problem of spurious reflections from model boundaries is overcome by using an impedance matching technique. This effectively assigns to the cells that need to be non-reflective on the boundaries of the model the properties of a massless viscous damper such that where  $\eta$  is the loss factor (dimensionless),  $K''$  is the imaginary

part of a complex spring stiffness in which the real part is zero,  $\omega$  the angular frequency,  $\rho c$  the characteristic impedance of the medium,  $\xi_0$  and  $\xi_{-1}$  are the displacements of cell points 0 and  $-1$  where the boundary is at cell 0,  $\rho$  is the density of the cell contents and  $v_0$  is the velocity of cell 0. Over 95% absorption is achieved across the spectrum. This is detailed as per the equation shown below-

$$\frac{\eta K'''}{\omega} = - \left( \rho c + \frac{D(\xi_0 - \xi_{-1})}{\rho \Delta x v_0} \Delta t \right)$$

### A.3 Input data

- A.3.1 The only input data required for the model is the masses of each cell, plus the shear modulus and the compression modulus, and the loss factor. Otherwise, all secondary parameters, such as wave speeds and impedances, are automatically generated by the finite difference algorithm. The only other input relates to methods of approximating actual structure shapes using the orthogonal grid.
- A.3.2 The output of the model consists of a file containing the displacement and/or velocity of one or more selected cells.
- A.3.3 The time steps used are of the order of 30 to 60 microseconds, and the model is run for either 16,384 or 32,768 steps to give a signal length of just under one second.
- A.3.4 The resulting discrete time series can then be subjected to discrete Fourier transformation to yield frequency spectra.
- A.3.5 Note that, whereas in the acoustical analogy the impedance of air varies little (except close to sources such as points), so that in most cases power is proportional to velocity squared, in elastic media, velocity transfer functions do not directly convey information about power transmission, and velocity at the receiver, in a low impedance medium, can be higher than velocity near the source, in a high impedance medium, even when there are power losses between the source and the receiver.

### A.4 Validation

- A.4.1 The finite difference algorithm is validated by creating models of structures for which algebraic solutions are available and comparing the eigenfrequencies and decay rates. For Timoshenko beams, plates, thin and thick cylinders, the eigenfrequencies are correctly predicted.

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Registered office Bridge House, 1 Walnut Tree Close, Guildford GU1 4LZ

National Highways Company Limited registered in England and Wales number 09346363